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Evaluation Different Sorting Criteria and Strategies Using Mathematical Programming

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Abstract

The pig industry is an essential and important part of Danish economy with an export value in 2006 of more than DKK 28 billions [Danish Meat Association (2007)]. The competition increases, and potential new competitors from low cost countries can be expected to enter the traditional Danish export markets. Therefore it is more important than ever to optimize all aspects of Danish pig production, slaughtering processes and delivery.

The raw material (the pigs) used by the slaughterhouses is a biological material, with a large variation in weight, size, fat layer and other quality characteristics. The slaughterhouses deal with this variation by sorting the pigs into different sorting groups, whereby variation within each sorting group is reduced substantially.

Deciding on the sorting criteria and sorting limits used to define the sorting groups has substantial influence on the economy of the slaughterhouses. In principle, sorting can be based on every kind of quality characteristics, e.g. fat layer, slaughter weight, lean meat percentage or whether the pigs are special production pigs (welfare pigs etc.). Each slaughterhouse defines its sorting groups according to customer demands (and whether or not a premium for that quality can be obtained) and the production costs (more uniform raw materials can make the production easier and less expensive).

The sorting parameters can be combined in different ways, and the sorting limits can have a numerous number of values. In this paper, the different aspects of sorting will be illustrated by using the following two sorting parameters:

- Fat layer (in mm) and
- Slaughter weight (in kg).

Figure 1 below is an example of sorting groups and sorting limits based on the fat layer.

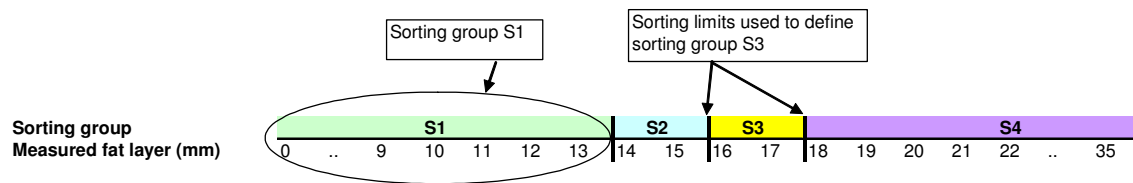


Figure 1. Example of sorting groups and limits for sorting based on fat layer.

The paper is concerned with tools for evaluation of different sorting strategies by the use of operations research methods. Evaluation of sorting strategies is a practical problem of major importance for the slaughterhouse industry. The model used is the same Mixed Integer Programming (MIP) model as described in the paper regarding limitations in production and stocks [Kjærsgaard, N. (2008c)].

The model is illustrated by performing experiments using slaughter data from 43,949 pigs slaughtered at one of the Danish slaughterhouses. In the computations, 17 different products and four alternative uses of each pig are used, but the model can easily be modified to include more products and alternative uses.

The main conclusion of the experiments is that even relatively simple optimization models can be used to improve the basis of the slaughterhouses for making decisions considerably. The graphical tool based on the optimization model provides an overview of the sorting criteria and limits which can be used to develop good sorting strategies, and the optimization model can be used to evaluate these different strategies further.

1 Background

The pig industry is essential for Danish economy and exports. In 2006, more than 25 million pigs were produced in Denmark, and approx. 90% of the meat was exported. The export value amounted to DKK 28.8 billion [Danish Meat Association (2007)].

Competition in the pig industry is substantial, and farmers from a number of countries can produce pigs at considerably lower costs than European farmers. Farmers from e.g. Brazil, USA and Canada are able to produce pigs at approximately 80% of the costs in Denmark, Spain and The Netherlands [Udesen, F. & Rasmussen, J. (2006)].

Even within our neighbouring countries there is a fierce competition for the slaughterhouses to offer the best payments to the farmers, and during the last couple of years a substantial number of Danish farmers have delivered part of their pigs to German slaughterhouses.

Therefore it is becoming more and more important that Danish farmers and slaughterhouses continue to optimize their production and slaughtering processes.

The slaughterhouses are characterized by the fact that the raw material (the pigs) is a biological material with a relative large variation in quality, size and shape. The slaughterhouses deal with this variation by sorting the pigs into different sorting groups. By so doing, the slaughterhouses are able to reduce the variation within each sorting group substantially.

This paper is concerned with the aspects of optimization at the slaughterhouses, specifically regarding computing of the value of improved sorting and evaluation of different sorting strategies. This practical problem is of major importance for the slaughterhouse industry and is solved by operations research methods.

2 Literature survey

The literature regarding optimized raw material use at the slaughterhouses has been addressed in the paper “The Value of Improved Measurements in a Pig Slaughterhouse” [Kjærsgaard, N. (2008a)] but is repeated here as it should be possible to read this paper independently.

The amount of literature addressing improved or optimized raw material use in the food industry is substantial. However, the main part of the contributions is related to different aspects regarding either optimization of meat quality or different production processes. Examples of this are optimization of the industrial thermal sterilization of canned foods [Garcia, M. et. al. (2006)] and pigs stunning optimization [Dupuis, P. et. al. (2004)]. These types of optimizations are not relevant for this project as they are either based on

statistical analysis without optimization of a mathematical model or the mathematical models are very different from the models, which are used in this Ph.D. project regarding optimization of the raw material use at the slaughterhouses.

Within the pork industry relatively few contributions have been found regarding optimization based on operations research methods. In the paper “Location of slaughterhouses under economies of scale” [Broek et. al. (2006)] optimization is used to investigate the savings potential of reducing the number of slaughterhouses in Norway and investing in additional capacity in the remaining facilities in order to obtain economies of scale. Another facility location problem is described in the paper “The impact of changes in livestock supply on the optimum number, size and location of slaughterhouses in East Macedonia” [Kamenidis, C. & Sorensen, V. (1978)]. In the paper “Economic optimization of pork production – marketing chains. II. Modelling outcome” [Ouden et. al. (1996)] are using Dynamic Linear programming to evaluate the development of pork chain concepts that also takes animal welfare into consideration. Kure in his Ph.D. thesis “Marketing Management Support in Slaughter Pig Production” [Kure, H. (1997)] uses Dynamic Programming to solve parts of the “slaughter pig marketing management problem”, which regards how the farmers should select and market their pigs to the slaughterhouses.

The above mentioned four examples of optimization problems within the pork industry are all somewhat different from the problem of optimizing the raw material use at the slaughterhouses. More similar problems have been found in the following contributions:

In 1990-1992 a project regarding optimization of the raw material use at the slaughterhouses was performed as a cooperation between Danish Meat Research Institute and the Royal Veterinary and Agricultural University (now the Faculty of Life Sciences at University of Copenhagen). Several reports were made:

A Linear Programming (LP) model for production planning and control for the hog slaughterhouses was developed and reported in [Rasmussen, S. & Thomsen, M. (1991)] and [Rasmussen, S. (1992)]. The model is a 2-stage model. First stage concerns a planning horizon of 3 months and the second stage one weeks day to day planning. In [Fertin, C. (1992)] the long term planning model (stage 1) is validated. In his Ph.D. thesis [Fertin [1995]] Fertin describes and further develops and validates the model.

There has been searched for literature in other food related industries, e.g. poultry and beef slaughterhouses and the fish industry, but no relevant literature has been found.

Other industries have similar problems as the slaughterhouses regarding its raw material use. An example is the refineries, but unlike the slaughterhouses the refineries have the option of blending different qualities in order to change the quality characteristics of the products. Another example is the lumber and wood industry. A few papers of the product mix problem within the wood industry have been identified. In the paper “An Optimization-Based Decision Support System for a Product Mix Problem” [Roy et. al. (1982)] an LP-model has been used to solve a plywood product mix problem for Ponderosa Industrial in Mexico.

Even though literature within food optimization is substantial, the main part of the contributions are related to optimization based on e.g. statistical analysis without optimization of a mathematical model. Other models are very different from the models used in this Ph.D. project. Except for the contributions from the Royal Veterinary and Agricultural University and the Danish Meat Research Institute not much literature of relevance for the Ph.D. project has been identified.

3 Sorting at the Slaughterhouses

The slaughterhouse industry is characterized by the fact that the raw material (the pigs) in the short term exists in a given volume with a relatively varied quality and has a limited shelf-life. The raw material can be used for several different products, but to a large extent yields, prices and costs depend on how well the raw material in question fits the final products.

The slaughterhouses deal with the natural variation in quality, weight, size, fat layer, lean meat percentage, etc. by sorting the pigs into different sorting groups, in which pigs with almost the same characteristics are placed. However, the variation within each sorting group is still substantial due to the considerable measuring error in the current measuring systems as well as the limited number of sorting groups which the slaughterhouses are able to handle.

For the years to come it is expected that the measuring accuracy will be improved substantially. This will result in more pigs being placed in correct sorting groups with less variation within each group as a consequence. If the full economic effect of improved sorting should be reached, more sorting groups are required.

In principle, sorting can be based on every kind of quality characteristics, such as the fat layer, lean meat percentage, slaughter weight, weight of a specific part, colour, pH-

value of the meat as well as whether the pigs are special production pigs (welfare pigs etc.). The sorting parameters chosen will depend on:

- Customer demands and whether a premium for the quality in question can be obtained and
- Production costs (more uniform raw materials can make production easier and less expensive).

Obviously, it is also important that the slaughterhouse can perform the measurements and that the logistics at the slaughterhouse are able to handle additional sorting groups.

In this paper, sorting aspects will be illustrated by using two quality characteristics: The fat layer and the slaughter weight. The possible sorting strategies investigated in this paper are:

1. Sorting based on slaughter weight (Figure 2a)
2. Sorting based on fat layer (Figure 2b)
3. Sorting based on fat layer and slaughter weight (Figure 2c)

The different strategies are illustrated in figure 2a-c.

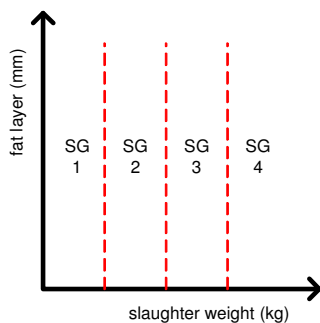


Figure 2a

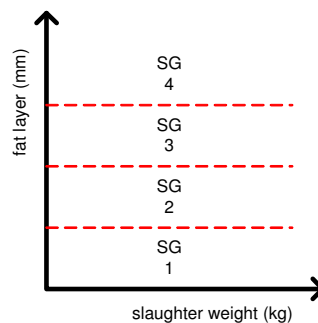


Figure 2b

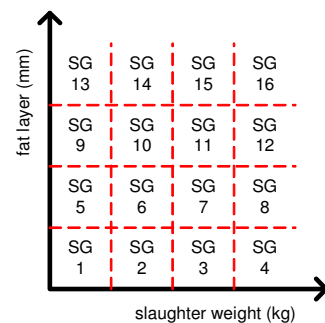


Figure 2c

Compared to other industries, the value added at the slaughterhouses is relatively limited. The Profit and Loss Account for 2006/07 for Tican a.m.b.a. [Tican (2007)] has been analysed in order to investigate the cost structure at the slaughterhouses. It was

found that approx. 65%¹ of the turnover is used for direct payments to the farmers for the pigs received and that only 35% are left to cover all the value adding activities taking place at the slaughterhouses. By investigating the cost structure it was found that even a small improvement in raw material use has a significant impact on profitability. If the raw material use for instance can be improved to increase the total yield by just 1%, this will increase the profit corresponding to:

- A decrease in administrative costs by 64%, or
- A decrease in the production wages by 7%.

These examples illustrate that one of the most important ways for the slaughterhouses to improve the earning power is to improve the raw material use. See [Kjærsgaard, N. (2008e)] chapter 2.1 for further information regarding the cost structure and value added at the slaughterhouses.

4 The Model

The model used is the same Mixed Integer Programming (MIP) model as described in the paper [Kjærsgaard N. (2008c)] regarding limitations in production and stocks which is based on the model developed in the paper regarding improved measurements [Kjærsgaard, N. (2008b)]. The model was described in these two papers, but as this paper should be readable independently it is repeated here. The benefits or costs are found by performing two optimizations, one under the current conditions and one under improved conditions. The improvement can then be found as the difference between the profits of the two optimizations.

4.1 Description of the Model

There are a number of logistic limitations at the slaughterhouses [Kjærsgaard, N. (2008c)]. The most important concerns the equalization room, where the temperature is equalized through the entire carcass. In the slaughterhouse used as basis for the modelling, the carcasses are placed in the equalization room on bars containing 80 carcasses each. Each bar can only be emptied from the opposite side of the filling side. In principle, carcasses

¹ Based on annual accounts for the parent company which include both the primary industry (slaughtering etc.) and the secondary industry (processed meat, sausages etc.). If only the primary industry is considered, the raw materials share of the turnover will increase further.

placed on the same bar are therefore used for the same production order (the same package of products).

In the experiments we use actual slaughtering data from 43,949 pigs slaughtered at one of the Danish slaughterhouses. For each pig the registered fat layer and the actual slaughter weight are used. In the computations the registered fat layer is considered to be the true value. When performing computations regarding improved measurements and sorting, a simulated measuring error is added to the registered fat layer and is then considered the measured fat layer.

The model has its basis in different alternative uses of the pigs. Each alternative use consists of a “package” of products for the specific part. The back and the ham have two alternative uses each and the fore end has one. In total there are four different alternative uses for each pig. In total, 17 different main products are used, but the model can easily be augmented with more products and alternative uses.

The different alternative uses for different parts of the pigs can be seen in Figure 3 below:

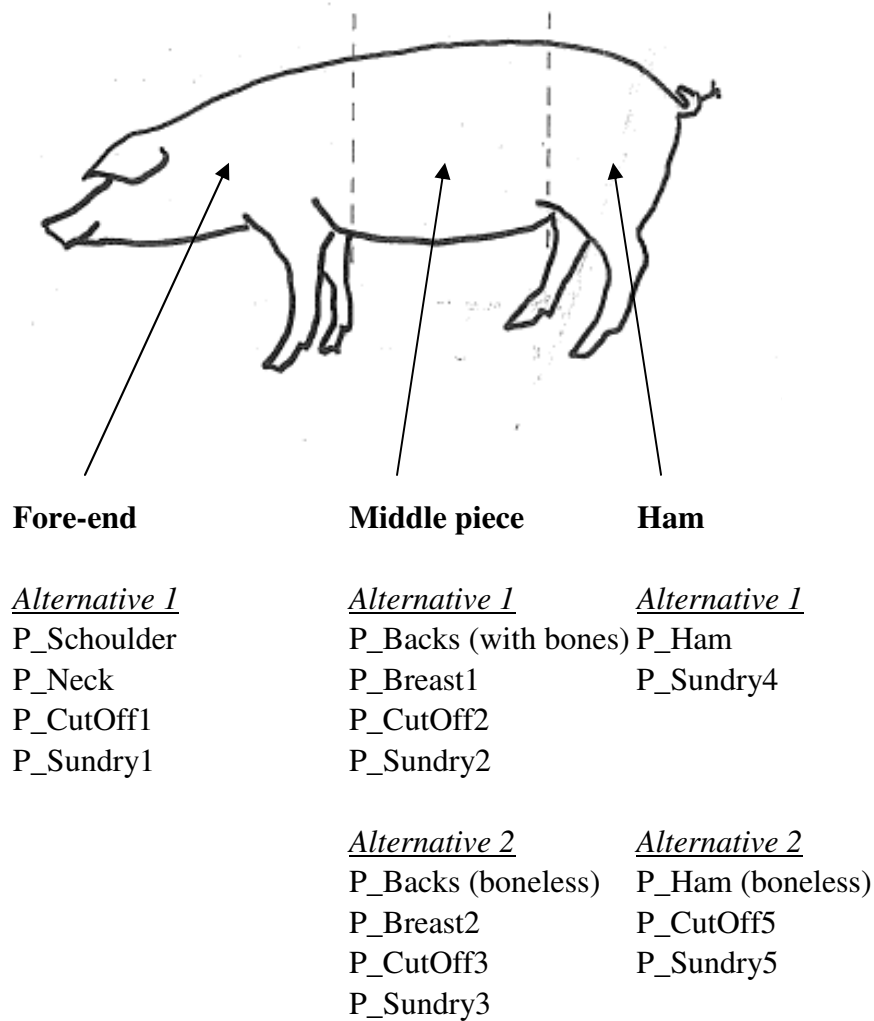


Figure 3. Alternative uses of the pig.

Some raw materials (pigs) are better suited for some products than others. This is taken into consideration when increasing or decreasing the price for some products depending on the level of the fat layer. In the model, this is done by splitting the pricing in two different contributions:

1. A fixed price per kg for the given product
- and

2. A price coefficient, which stipulates how much the price will decrease if the fat layer increases by 1 mm.

For two of the products there are a few special conditions which have to apply for the raw materials to be used for these products:

- The ham product P_Ham can only be produced if the fat layer does not exceed 14 mm

and
- The breast product P_Breast2 can only be produced if it does not exceed a weight of 4 kg.

If these conditions are not met, a penalty is introduced in the prices. The penalty covers additional handling costs if these raw materials should be used for other products instead.

The products P_CutOff (1, 2, 3, 5) consist of meat cut-offs in connection with production of the main products, and products P_Sundry (1-5) consist of fat, bones, rind etc.

4.2 Mathematical Formulation of the Model

We have a set of carcasses $\mathcal{I}=\{1,...,I\}$. Each carcass can be used to produce a set of different product alternatives $\mathcal{N}=\{1,...,N\}$ and each product alternative consists of a set of different products $\mathcal{J}=\{1,...,J\}$. Finally the carcasses are hung on a set of bars $\mathcal{K}=\{1,...,K\}$ in the equalization room. The decision variable $y_{k,n}$ is a binary variable with the value 1 if the pigs placed on bar k are used to produce product alternative n and otherwise 0. The problem is to find the optimal utilization (product alternatives) of the carcasses placed at each bar and the total profit for the optimal solution:

The objective function:

$$1) \quad \text{Maximize } Z = \sum_{k,n} \text{ValueBar}_{k,n} \bullet y_{k,n}$$

Subject to:

$$2) \quad \sum_n y_{k,n} = 1 \quad \forall k \in \mathcal{K}$$

$$3) \quad y_{k,n} = \{1 \text{ if product alternative } n \text{ is produced by pigs placed on bar } k, \text{ otherwise } 0 \}$$

$$4) \quad ValuePig_{i,n} = \sum_{i,n} (Price_j + PriceCoeff_j \bullet FatLayerDeviation_i - QualityDeduction_{i,j}) \bullet ProdWeight_{i,j} \bullet AltUse_{j,n} \\ \forall \quad i \in \mathcal{I}, n \in \mathcal{N}$$

$$5) \quad ValueBar_{k,n} = \sum_k ValuePig_{k,n} \quad \forall \quad k \in \mathcal{K}, n \in \mathcal{N}$$

Indices:

i : pig i k : bar k j : product j n : alternative use n

Decision variables:

$y_{k,n}$: Decision variable with value 1 if the carcasses placed on bar k are used for product alternative n , otherwise 0.

Parameters:

$ValueBar_{k,n}$: Value of the carcasses placed on bar k when used to produce alternative n .

$ValuePig_{i,n}$: Value of carcass i , when producing alternative n .

$Price_j$: Fixed net price per kg for producing product j .

$PriceCoeff_j$: Change in net price per kg for product j when the fat layer increases by 1 mm.

$FatLayerDeviation_i$: Deviation in the fat layer of carcass i compared to the average fat layer.

$QualityDeduction_{i,j}$: Price deduction per kg if quality demands are not being met when carcass i is used for production of product j .

$ProdWeight_{i,j}$: Estimated weight of product j , when produced from carcass i .

$AltUse_{j,n}$: Alternative use (product package) with value 1 if product j is part of product alternative n , otherwise 0.

The objective function (1) maximizes the sum of the value of carcasses at each bar by finding the best alternative use for each bar when all pigs placed on the same bar are used for the same product alternative. The constraint (2) controls that the carcasses placed at each bar are only used once. The model uses a number of different parameters. The most important ones are shown in (4) and (5) and are either directly or indirectly used in the objective function. The parameter $ValuePig_{i,n}$ (4) finds the value of each carcass i , when producing product alternative n . The value is based on a price per kg for each potential product, a price coefficient depending on the fat layer and a deduction in price if certain quality measurements are not met. This net price for different products is multiplied with the estimated weight of the products. The parameter $ValueBar_{k,n}$ finds the total value of carcasses placed at bar k , when producing product alternative n .

5 Results

As mentioned before, the sorting strategies illustrated in this chapter are relatively simple strategies as they are using only two sorting criteria: Slaughter weight and fat layer. This provides the following three main principles for sorting:

1. Sorting based on slaughter weight
2. Sorting based on fat layer
3. Sorting based on both fat layer and slaughter weight

5.1 Sorting based on slaughter weight

The 43,949 pigs used in the experiments have been sorted into four different sorting groups based on the registered slaughter weight. The four sorting groups contain almost the same number of pigs and are used to place the carcasses on bars. The distribution of pigs on slaughter weight and sorting groups can be seen in Figure 4 below.

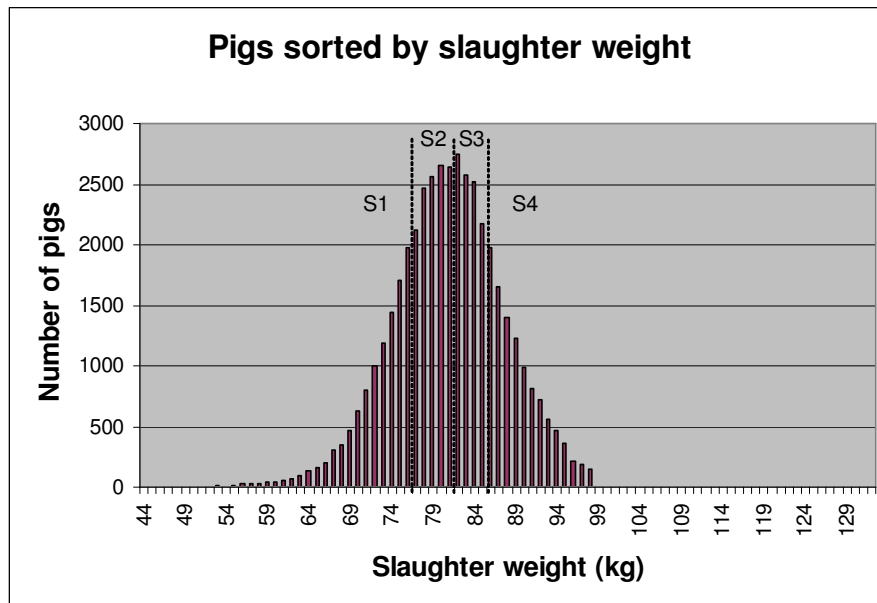


Figure 4. Distribution of pigs on slaughter weight and sorting groups.

The optimal use of the pigs has been found provided that all pigs placed on the same bar are used for the same product alternative. The profit of the 43,949 pigs used in the experiment is computed to:

DKK	
Profit	37,810,962

Table 1. Result of experiment with four sorting groups based on slaughter weight

Compared to no sorting at all, where pigs are placed on bars without taking any quality measurements into consideration this is an improvement in profit by DKK 80,167.

The sorting strategy could be slightly changed by using other limits for slaughter weights used to define each of the four sorting groups. Furthermore, the number of sorting groups could be increased as well.

5.2 Sorting based on fat layer

Now the fat layer of the 43,949 pigs used in the experiments has been simulated, and the pigs are sorted based on these simulated values of the measured fat layer. All four sorting groups are of approximately the same size and are used when placing the carcasses on bars. The distribution of pigs on measured fat layer at the current level of measuring accuracy and on the sorting groups can be seen in Figure 5 below.

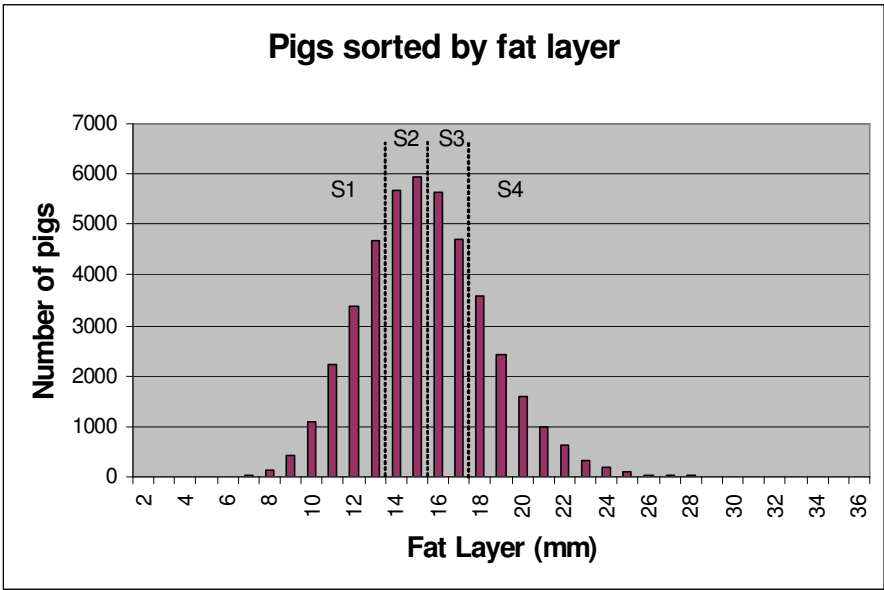


Figure 5. Distribution of pigs on measured fat layer at the current level of measuring accuracy and on the sorting groups.

When using the bars optimally the profit of the 43,949 pigs is computed to:

DKK	
Profit	37,827,885

Table 2. Profit with sorting based on fat layer at the current level of measuring accuracy.

Compared to sorting based on slaughter weight alone, the profit increases by an additional DKK 16,924 for the 43,949 pigs being part of the experiment when both computations are based on four sorting groups of approximately the same size.

The measuring system's ability to measure accurately is specified by its standard error of prediction (SEP). The standard error of prediction is found as the standard deviation of the differences between the measured values and the reference values (true values) using a test data set. For each level of measuring accuracy a similar distribution as the one in Figure 5 is estimated and sorting limits for the sorting groups are established. The carcasses are placed on bars based on sorting groups. Computations of the optimal use of carcasses are made, and the profit for different levels of measuring accuracy is found and can be seen in Table 3 below:

	Profit	Improved profit
Current measuring error (SEP)	37,827,885	16,924
Current measuring error (SEP) - 20%	37,870,426	59,464
Current measuring error (SEP) - 40%	37,916,962	106,000
Current measuring error (SEP) - 50%	37,941,708	130,746
Current measuring error (SEP) - 60%	37,966,898	155,936
Current measuring error (SEP) - 80%	38,027,786	216,824
Current measuring error (SEP) - 100%	38,066,957	255,996

Table 3. Improved profit with sorting based on measured fat layer compared to sorting by weight

It can be seen that when the measuring accuracy is improved, the profit increases substantially. The improvement in profits is almost linear with approximate DKK 2,400 for each percentage the measuring accuracy (SEP) is improved.

5.3 Sorting based on both slaughter weight and fat layer

Now the sorting is based on both the slaughter weight and the fat layer. The sorting is still based on the same sorting limits, which were used previously, but now it requires 16 sorting groups instead of four (see Figure 6 below).

	slaughter weight (kg)																																				
	0-63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99
fat layer (mm)	0-7	SG 1														SG 2					SG 3					SG 4											
	8																																				
	9																																				
	10																																				
	11	SG 5														SG 6					SG 7					SG 8											
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Figure 6. Sorting groups and sorting limits.

The profit increases substantially when sorting is based on both fat layer and slaughter weight. Compared to the scenario where sorting is based on the fat layer alone, the profit at the current measuring accuracy increases by DKK 104,064 for the 43,949 pigs being part of the experiment. This equals DKK 59 million for the Danish slaughterhouses on an annually basis. The profit improvements at different levels of the measuring error can be seen in table 4 below.

	Profit	Improved profit
Current measuring error (SEP)	37.931.949	104.064
Current measuring error (SEP) - 20%	37.964.185	136.300
Current measuring error (SEP) - 40%	38.001.568	173.683
Current measuring error (SEP) - 50%	38.024.909	197.024
Current measuring error (SEP) - 60%	38.048.667	220.782
Current measuring error (SEP) - 80%	38.099.793	271.908
Current measuring error (SEP) - 100%	38.148.722	320.837

Table 4. Profit with sorting based on both slaughter weight and fat layer compared to sorting based on fat layer alone.

Based on the figures in Table 4, which stems from computations for the 43,949 pigs used in the experiment, the consequences of improved measurements have been calculated. For the Danish slaughterhouses, which produce approximately 25 million pigs per year, the equivalent improvement in profits can be seen in Figure 7 below:

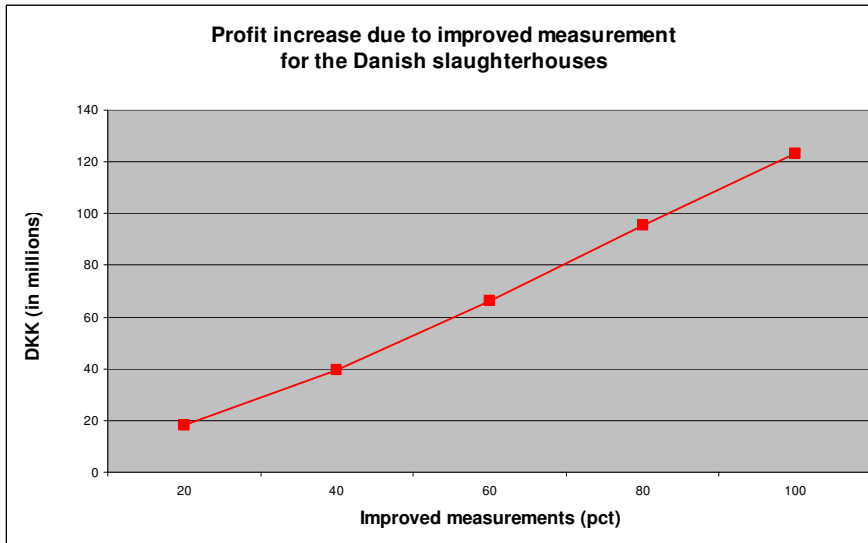


Figure 7. Profit increase for the Danish slaughterhouses due to improved measurements with sorting based on both slaughter weight and fat layer.

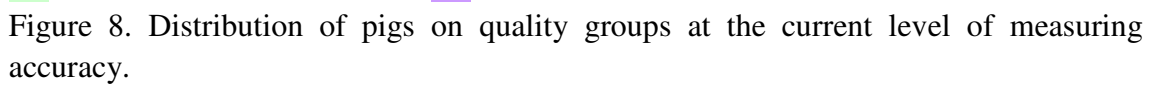
Figure 7 shows that improved measurements are valuable for Danish slaughterhouses. If the measurements were perfect (current measuring error reduced 100%), the increased profits for Danish slaughterhouses is estimated to more than DKK 120 million per year.

Even though the profit increases substantially by using the 16 sorting groups indicated in Figure 6, it is possible to increase the profit further by defining the sorting limits in a more intelligent way than just requiring them to be of approximately the same size. This may even be possible with much fewer sorting groups.

The GAMS code can be seen in Appendix 1. The solution time for solving the model to optimality was just 17 seconds and this is considered very acceptable. See chapter 4.2 in the thesis [Kjærsgaard, N. (2008e)] for further information regarding solution times when increasing the number of products and product alternatives.

5.4 Definition of sorting groups

The 43,949 pigs used in the experiments are divided into 20 quality groups according to the fat layer and 37 groups according to slaughter weight. This in total gives 740 combinations or quality groups. The distribution of pigs in different quality groups can be seen in the matrix in Figure 8 below:



Sorting group A:

Sorting group B:

Sorting group C:

Sorting group D:

The four more “intelligent” sorting groups can be seen in Figure 9 below.

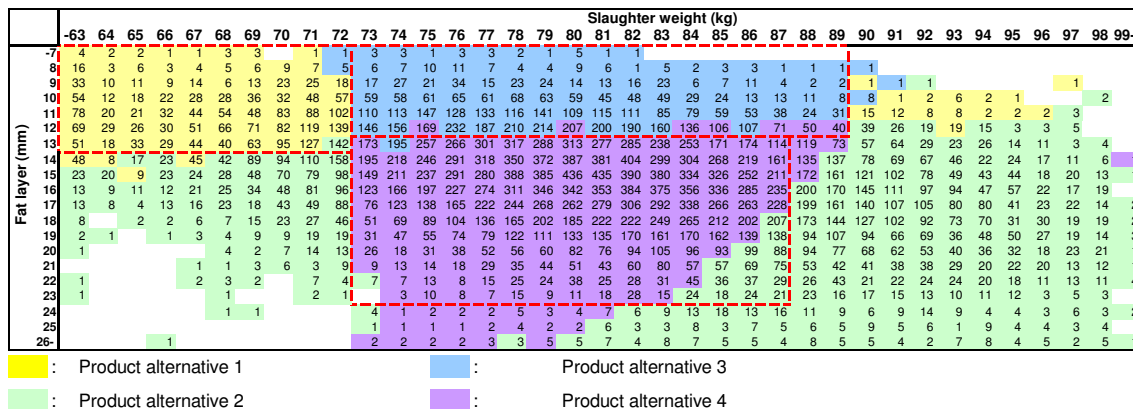


Figure 9. Distribution of pigs on quality groups at the current level of measuring accuracy with improved sorting groups.

Figure 9 indicates that this new sorting strategy is a clear improvement. Sorting group A (in the upper left corner) mostly consists of quality groups where the optimal production is product alternative 1 (marked yellow). Sorting group B seems quite good too, but it might be possible to improve sorting group C and D further. The profit using the sorting groups, which can be seen in Figure 9 above, is computed to:

Profit	38.013.205

Table 5. Profit using new sorting criteria.

This is a further improvement in profits by DKK 81,256 for the 43,949 pigs used and equals DKK 46 million for the Danish slaughterhouses on an annually basis. If each of the 740 quality groups of which the matrix in Figure 10 consist were used optimally, the profit can be computed to DKK 38,039,658 for the 43,949 pigs used.

When using the four sorting groups A-D the profit should be compared to the computed profit using the 740 quality groups, as this can be considered an upper bound for the size of the profit. The profit is only improved by DKK 26,453, equalling DKK 15 million annually for the Danish slaughterhouse when using 740 sorting groups instead of four (A-D).

As suggested, the sorting can be improved even further by using the following five sorting groups instead:

Sorting group A1:

slaughter weight < 73

fat layer < 14

Sorting group B: $73 \leq$ slaughter weight < 90

fat layer < 13

Sorting group C: $73 \leq$ slaughter weight < 90 $13 \leq$ fat layer

if fat layer < 26:

slaughter weight + fat layer ≤ 104 if fat layer ≥ 26 :

slaughter weight < 80

Sorting group D: $90 \leq$ slaughter weight < 97 $10 \leq$ fat layer < 12**Sorting group E:**

Rest

Sorting group C is a bit more complex as there are different criteria whether or not the fat layer is less than 26 mm. The five sorting groups can be seen in Figure 10 below but only result in minor improvements. The number of carcasses without similarity in the optimal use of sorting group and quality group make up 5.0% using the new sorting groups indicated in Figure 10 and 6.4% using the previous sorting groups from Figure 9.

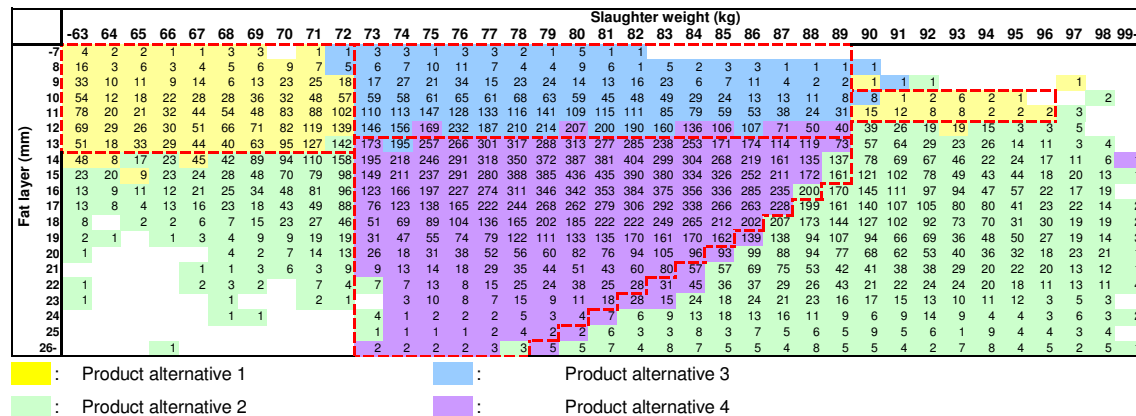


Figure 10. Distribution of pigs on quality groups using 5 sorting groups.

6 Conclusion

Even the very simple sorting strategy using only four sorting groups of approximately the same size and pigs sorted based only on slaughter weight, the computed profit is increased by DKK 80,167 compared to no sorting for the 43,949 pigs used in the experiment. This equals DKK 46 million for Danish slaughterhouses on an annually basis.

When a similar sorting in four sorting groups of approximately the same size is used, but this time based on the fat layer instead of the slaughter weight, the profit increases by an additional DKK 16,924 with the current level of measuring accuracy. For the Danish slaughterhouses this equals an additional DKK 10 million annually.

When combining sorting based on slaughter weight and on fat layer the profit improves substantially. When using the same sorting criteria as when the sorting was based on slaughter weight and on fat layer individually, this 2 dimensional sorting strategy requires 16 sorting groups. Using this strategy, the profit improves with an additional DKK 104,064 which equals DKK 59 million on an annually basis for Danish slaughterhouses.

The profit increases substantially when the accuracy of the measurements improves. Computations have been performed; still using sorting based on the 16 sorting groups mentioned in Figure 6 above, and gives the following increase in profits for the Danish slaughterhouses on an annually basis:



Figure 11. Increased profit due to improved measurements for Danish slaughterhouses.

If the measurements were perfect (current measuring error reduced 100%), the increased profits for Danish slaughterhouses is estimated to more than DKK 120 million per year.

It has been shown that the model can be used to evaluate and compare different sorting strategies. The matrix which can be seen in Figure 12 below is an important tool in connection with designing new sorting strategies, as it provides a graphical view of the potential sorting groups worth investigating further.

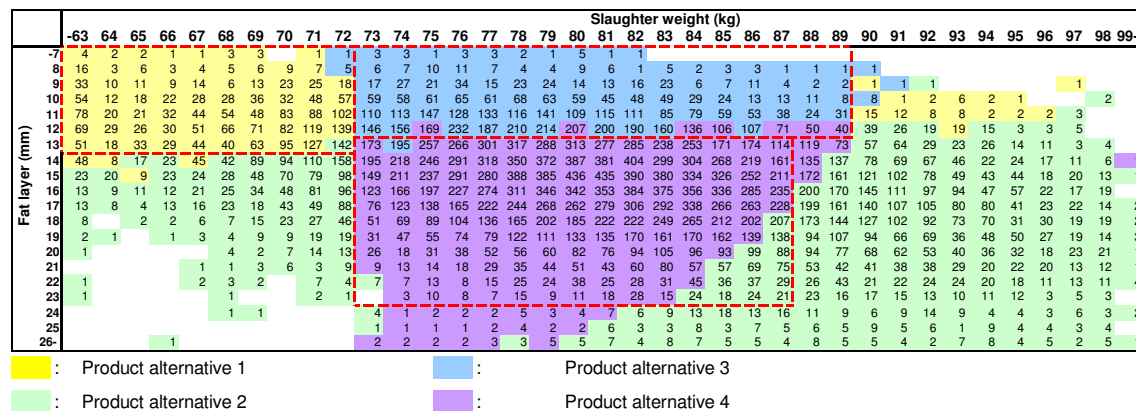


Figure 12. Distribution of pigs on quality groups at the current level of measuring accuracy with improved sorting groups.

The main conclusion is that even relatively simple optimization models can advantageously be used to improve the basis of the slaughterhouses for making decisions considerably. The graphical tool based on the optimization model provides an overview of the sorting criteria and limits which result in good sorting strategies, and the optimization model can be used to evaluate these different strategies further.

6.1 Future work

Before the slaughterhouses can rely on the model for actual decision making, several products and product alternatives should be included as input in the model, and a price and cost study should be obtained.

In this paper, the simulation of which carcasses are placed at different bars has been made in Excel outside the optimization environment GAMS². If a sub program is made to perform this task within GAMS or a similar optimization environment new simulations can be performed fast.

² GAMS (The General Algebraic Modeling System) is a high-level modelling system for mathematical programming problems.

A graphical interface based on the matrix in Figure 13 where sorting groups etc. are defined graphically could be a very interesting tool for the slaughterhouse. Further work should be made to find out how best to represent more dimensions than the current two: Slaughter weight and fat layer.

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Appendix 1 – GAMS code

```

* CanneryTransport.gms
*
* CanneryTransport.gms
*
$eolcom //
option iterlim=999999999; // avoid limit on iterations
option reslim=300; // timelimit for solver in sec.
option optcr=0.0; // gap tolerance
option solprint=OFF; // include solution print in .lst file
option limrow=100; // limit number of rows in .lst file
option limcol=100; // limit number of columns in .lst file
//-----

SETS
  i  Pigs i / p1*p43949/
  j  Products / P_Schoulder, P_Neck, P_Backs (with bones), P_Breast1, P_Backs (boneless),
              P_Breast2, P_Ham, P_Ham (boneless), P_CutOff1, P_CutOff2,
              P_CutOff3, P_CutOff5, P_Sundry1, P_Sundry2, P_Sundry3,
              P_Sundry4, P_Sundry5, P_moerbrad, P_hoved, H_8201 /
  s(j) Products sold / P_Schoulder, P_Neck, P_Backs (with bones), P_Breast1, P_Backs (boneless),
                      P_Breast2, P_Ham, P_Ham (boneless), P_CutOff1, P_CutOff2,
                      P_CutOff3, P_CutOff5, P_Sundry1, P_Sundry2, P_Sundry3,
                      P_Sundry4, P_Sundry5, P_Tenderloin, P_Head /
  l  Bar l / Bar1*Bar585 /
  n  Product Alternative n / Alt1*Alt4 /

$Include weight_5_SG100_fordeling_stænger_65SG.txt
;

PARAMETER
  Price(j) Price per kg for products j
  /
    P_Schoulder 12.00
    P_Neck 13.00
    P_Backs (with bones) 18.00
    P_Breast1 13.00
    P_Backs (boneless) 25.00
    P_Breast2 17.00
    P_Ham 15.00
    P_Ham (boneless) 18.00
    P_CutOff1 9.00
    P_CutOff2 9.00
    P_CutOff3 9.00

```

P_CutOff5	9.00
P_Sundry1	3.00
P_Sundry2	3.00
P_Sundry3	3.00
P_Sundry4	3.00
P_Sundry5	3.00
P_Tenderloin	30.00
P_Head	3.00
H_8201	0.00

/

PriceCoeff(j) Price Coefficient (in DKK) for product j for an increase of layer of fat (in mm)

/ P_Soulder	0.00
P_Neck	0.00
P_Backs (with bones)	-0.20
P_Breast1	-0.20
P_Backs (boneless)	-0.20
P_Breast2	-0.20
P_Ham	-0.20
P_Ham (boneless)	-0.20
P_CutOff1	-0.10
P_CutOff2	-0.10
P_CutOff3	-0.10
P_CutOff5	-0.10
P_Sundry1	0.00
P_Sundry2	0.00
P_Sundry3	0.00
P_Sundry4	0.00
P_Sundry5	0.00
P_Tenderloin	0.00
P_Head	0.00
H_8201	0.00

/

ProdWeightCon(j) Product weight constant for product j

/ P_Soulder	0.00000
P_Neck	0.00000
P_Backs (with bones)	10.77058
P_Breast1	2.00642
P_Backs (boneless)	0.46036
P_Breast2	2.00642
P_Ham	0.00000
P_Ham (boneless)	-1.11490
P_CutOff1	0.00000
P_CutOff2	0.00000
P_CutOff3	0.00000
P_CutOff5	0.00000
P_Sundry1	-1.95414

P_Sundry2	-14.54192
P_Sundry3	0.00000
P_Sundry4	0.00000
P_Sundry5	0.00000
P_Tenderloin	1.20000
P_Head	0.00000
H_8201	-1.58570

/

ProdWeightFat(j) Product weight fat dependend coefficient for product j

/ P_Schoulder	-0.06938
P_Neck	-0.04096
P_Backs (with bones)	-0.01662
P_Breast1	0.04284
P_Backs (boneless)	- 0.08124
P_Breast2	0.04284
P_Ham	-0.10204
P_Ham (boneless)	-0.19054
P_CutOff1	-0.00596
P_CutOff2	-0.00596
P_CutOff3	-0.00596
P_CutOff5	-0.00596
P_Sundry1	0.07922
P_Sundry2	0.11178
P_Sundry3	0.00000
P_Sundry4	0.00000
P_Sundry5	0.00000
P_Tenderloin	0.00000
P_Head	0.00000
H_8201	-0.10160

/

\$Include FatLayer_beg.txt

\$Include PigWeight_beg.txt

ProdWeightWeight(j) Product weight slaughtering weight dependent coefficient for product j

/ P_Schoulder	0.10726
P_Neck	0.07282
P_Backs (with bones)	0.01354
P_Breast1	0.06002
P_Backs (boneless)	0.08666
P_Breast2	0.06002
P_Ham	0.27632
P_Ham (boneless)	0.22874
P_CutOff1	0.00834
P_CutOff2	0.00834

P_CutOff3	0.00834
P_CutOff5	0.00834
P_Sundry1	0.13368
P_Sundry2	0.24410
P_Sundry3	0.00000
P_Sundry4	0.00000
P_Sundry5	0.00000
P_Tenderloin	0.00000
P_Head	0.00000
H_8201	0.29790

/

;

Table Anvendelse(j,n) Product alternative n in which product j is part of

	Alt1	Alt2	Alt3	Alt4
P_Schoulder	1	1	1	1
P_Neck	1	1	1	1
P_Backs (with bones)	1	1	0	0
P_Breast1	1	1	0	0
P_Backs (boneless)	0	0	1	1
P_Breast2	0	0	1	1
P_Ham	1	0	1	0
P_Ham (boneless)	0	1	0	1
P_CutOff1	1	1	1	1
P_CutOff2	1	1	0	0
P_CutOff3	0	0	1	1
P_CutOff5	0	1	0	1
P_Sundry1	1	1	1	1
P_Sundry2	1	1	0	0
P_Sundry3	0	0	1	1
P_Sundry4	1	0	1	0
P_Sundry5	0	1	0	1
P_Tenderloin	1	1	1	1
P_Head	1	1	1	1
H_8201	0	0	0	0

;

Parameter ProdWeight(j,i) Weight of product j from pig i ;

ProdWeight(j,i) = ProdWeightCon(j) + ProdWeightFat(j)*FatLayer(i) +
ProdWeightWeight(j)*PigWeight(i) ;

ProdWeight('P_Sundry3',i) = ProdWeight('P_Backs (with bones)',i) + ProdWeight('P_Breast1',i) +
ProdWeight('P_CutOff2',i)
+ ProdWeight('P_Sundry2',i) - ProdWeight('P_Backs (boneless)',i)
- ProdWeight('P_Breast2',i) - ProdWeight('P_CutOff3',i) ;

$\text{ProdWeight}('P_Sundry4',i) = \text{ProdWeight}('H_8201',i) - \text{ProdWeight}('P_Ham',i) ;$

$\text{ProdWeight}('P_Sundry5',i) = \text{ProdWeight}('H_8201',i) - \text{ProdWeight}('P_Ham (boneless)',i) - \text{ProdWeight}('P_CutOff5',i) ;$

$\text{ProdWeight}('P_hoved',i) = \text{PigWeight}(i) - \text{ProdWeight}('P_Shoulder',i) - \text{ProdWeight}('P_Neck',i) - \text{ProdWeight}('P_Bacs (with bones)',i) - \text{ProdWeight}('P_Breast1',i) - \text{ProdWeight}('P_Ham (boneless)',i) - \text{ProdWeight}('P_CutOff1',i) - \text{ProdWeight}('P_CutOff2',i) - \text{ProdWeight}('P_CutOff5',i) - \text{ProdWeight}('P_Sundry1',i) - \text{ProdWeight}('P_Sundry2',i) - \text{ProdWeight}('P_Sundry5',i) - \text{ProdWeight}('P_Tenderloin',i) ;$

Parameter $\text{QualityDeduction}(j,i)$ Deduction in price at product weight above 3.5 kg per back (7 kg per pig) and requirement for breast and ham ;

$\text{Fradrag}('P_Bacs (with bones)',i) = 2\$(\text{ProdWeight}('P_Bacs (with bones)',i) \text{ gt } 7) + 0\$(\text{ProdWeight}('P_Bacs (with bones)',i) \text{ le } 7) ;$

$\text{Fradrag}('P_Bacs (boneless)',i) = 2\$(\text{ProdWeight}('P_Bacs (boneless)',i) \text{ gt } 7) + 0\$(\text{ProdWeight}('P_Bacs (boneless)',i) \text{ le } 7) ;$

$\text{Fradrag}('P_Breast2',i) = 6\$(\text{ProdWeight}('P_Breast2',i) \text{ gt } 8) + 0\$(\text{ProdWeight}('P_Breast2',i) \text{ le } 8) ;$

$\text{Fradrag}('P_Ham',i) = 4\$(\text{FatLayer}(i) \text{ gt } 14) + 0\$(\text{FatLayer}(i) \text{ le } 14) ;$

Parameter $\text{ValueGris}(i,n)$ Price for pig i at alternative use n ;

$\text{ValueGris}(i,n) = \text{sum}(j, (\text{Price}(j) - \text{Fradrag}(j,i) + \text{PriceCoeff}(j) * (\text{FatLayer}(i) - 15.9)) * \text{ProdWeight}(j,i) * \text{Anvendelse}(j,n)) ;$

Parameter $\text{ValueStang}(l,n) ;$

\$Include ValueStang_43949.txt

Variables

z total profit

Binary Variables

y(l,n) 1 if alternative n is chosen to be produced of pig i with bars;

// y(i,n) 1 if alternative n is chosen to be produced of pig i without bars;

Equations

profit definering af objekt funktion

```

        con(l)    * with bars
//      con1(i)  * without bars
        ;

        profit ..  z =e= sum((l,n), ValueBar(l,n)*y(l,n)) ;
//      profit ..  z =e= sum((i,n), ValuePig(i,n)*y(i,n)) ;

        con(l) ..  sum((n),y(l,n)) =e= 1 ;
//      con1(i) ..  sum(n,y(i,n)) =e= 1 ;

```

```

Model begraensninger_v11 /all/ ;
Solve begraensninger_v11 using mip maximizing z ;

```